GNSS Signal Simulation and a Multipath Delay Estimation

Marko S. Djogatović, Milorad J. Stanojević

Abstract - In the satellite navigation systems, distortion of a received signal correlation function, due to the multipath propagation, can gravely degrade position estimation. The positioning accuracy is strongly affected by the quality of the received signal time-delay estimations. In the paper, signal and channel models for the L1 channel GPS C/A signal and the Galileo BOC(1,1) signal will be presented and the multipath mitigation problem analyzed. In addition, the MEDLL algorithm and the particle filter will be presented in detail and mutually compared for different simulated signals and different correlation times.

Keywords - Multipath, Signals, Simulation, Estimation, GNSS.

I. Introduction

In modern Global Satellite Navigation Systems (GNSS), the most significant source of error that affects the navigational signals during their propagation is multipath [1]. Multipath degrades the correlation function in such a way that it is not possible to determine accurately the signal delay. Therefore, removal of multipath is very important for applications where high precision measurements are required (geodesy and surveying, instrument landing systems, atmospheric sensing) or for indoor positioning where line-of-sight (LOS) signal is highly deteriorated by multipath. In some applications, like remote sensing, removing influence of multipath replicas is not enough. Hence, it is necessary to determine amplitude and time-delay of these replicas.

So far, a various multipath mitigation methods have been developed. Fig. 1 represents hierarchy of commonly used multipath mitigation techniques. Many of these methods are using correlation of early and late signal replicas with a received signal in order to find value of the time-delay that corresponds to the maximum power of correlation. Narrow early-minus-late (EML) correlation technique is derived from standard EML by narrowing 0.5 chip space between early and late correlators to 0.1-0.2 chip space. [2,3]. Other important correlator-based multipath mitigation techniques are: Double Delta (ΔΔ), also known as High Resolution Correlator (HRC) [4], then Strobe and Enhanced Strobe Correlator (ESC) [5], E1/E2 Tracker [6], Multipath Elimination Technique (MET) known as Early Late Slope (ELS) with Pulse Aperture Correlator (PAC) as a simple hardware implementation [7,8] and efficient technique based on Teager-Kaiser operator (TK) [9].

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Maximum Likelihood (ML) estimation is based on the maximum likelihood principle and is very popular approach in signal processing. Once a signal model is specified with its parameters, and data have been collected, the maximum likelihood estimator is used to find the value of parameter that maximizes likelihood function. So far, several maximum likelihood estimation techniques have been used for multipath mitigation: Newton methods with analytical [10,11] and numerical (FIMLA, RML) [12,13] expressions for gradient and Hessian term, then, the most famous ML method, Maximum Estimating Delay Lock Loop (MEDLL) [14,15] with its modifications (ML2 and Reduced Search Space ML - RSSML) [16,17], Multipath Mitigation Technique (MMT) integrated into Novatel's Vision Correlator [18] and iterative methods based on expectation-maximization algorithms (Space Alternating Generalized Expectation-Maximization – SAGE) [19].

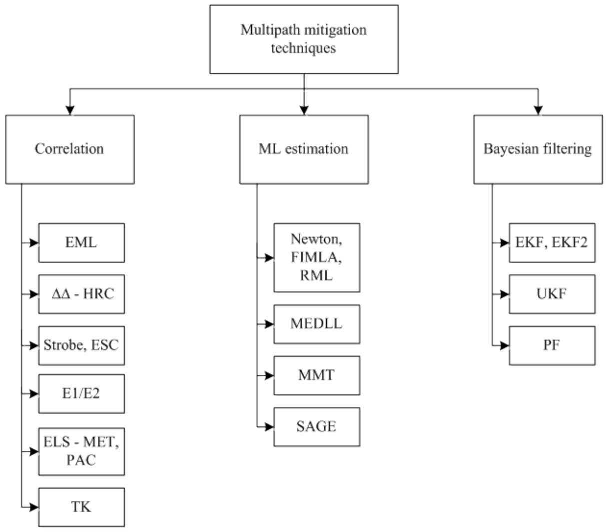

Fig. 1. Hierarchy of multipath mitigation techniques

Bayesian filtering is concerned with the estimation of the underlying probability distribution of a random signal in order to extract the original signal from noisy measurements. In order to mitigate influence of the reflected signal components on LOS signal delay estimation following Bayesian filters have been used: Extended Kalman Filter (EKF) [20,21] and Second-order Extended Kalman Filter (EKF2) [21], Unscented Kalman Filter (UKF) [22] and Particle Filter (PF) [23,24].

In the paper, an emitted and a received signal models for the GPS L1 C/A (coarse/acquisition) signal and the Galileo BOC(1,1) (Binary Offset Carrier) signal will be presented and the multipath mitigation problem investigated. Moreover, two estimation algorithms will be presented in detail and analyzed: the well-established and efficient MEDLL algorithm in contrast to the newly developed particle filter method.

II. SIGNAL AND CHANNEL MODEL

A. Transmitted signal

The signal $s(t)$ transmitted from one satellite can be written as [3,17]

$$s(t) = \sqrt{E_b} \cdot q(t) \cdot \cos 2\pi f_1 t \,, \tag{1}$$

where E_b is the bit energy, $q(t)$ is the navigation data after spreading and f_1 is the L1 carrier wave frequency. Spreading of navigation data bits, $\{d(n)\}$, is done as

$$q(t) = \sum_{n=-\infty}^{\infty} d(n)\, p(t - nT_b), \tag{2}$$

where $p(t)$ is the spreading waveform and T_b is the period of one data bit of the navigation message. The spreading waveform can be written as follows [17]

$$p(t) = g(t) \star \sum_{k=0}^{N_c - 1} c(k)\, \delta(t - kT_c). \tag{3}$$

Above, $g(t)$ is the modulation waveform (GPS L1 C/A signal or composite BOC(–) for Galileo E1-C signal), $\delta(\cdot)$ is the Dirac delta function, $\{c(k)\}$ is the spreading, pseudorandom (PRN) sequence of length N_c and the star sign ($\star$) denotes convolution. T_c is duration of one chip in the code sequence.

The modulation waveform, $g(t)$, can be written as

$$g(t) = g_P(t) \star \sum_{i=0}^{N_{sw} - 1} \delta(t - iT_{sw}), \tag{4}$$

where N_{sw} is modulation order (the number of periods of the square wave within one chip), $T_{sw} = T_c / N_{sw}$ is the period of square wave, and $g_P(t)$ is the shaping pulse. For GPS C/A signal is true that $N_{sw} = 1$ ($g(t) = g_B(t)$) and for BOC modulation that $N_{sw} = 2f_{sc} / f_c$, where $2f_{sc}$ is the is the square wave frequency and f_c is the chip frequency [17].

The shaping pulse $g_P(t)$ can be defined as filtered rectangular pulse using following equation

$$g_P(t) = \frac{1}{\pi T_{sw}} \left[Si(2\pi b t / T_{sw}) - Si\left(2\pi b (t / T_{sw} - 1)\right) \right], \tag{5}$$

where b describes the location of the cut-off frequency and it is related to the bandwidth, B_w, through the relation $b = B_w T_{sw} / (2\pi)$. $Si(\cdot)$ is the sine integral. On Fig. 1 a) and b) are shown the GPS C/A pulse and the Galileo BOC(1,1) pulse, respectively, for infinite bandwidth and in the band-limited case (B_w = 6 MHz).

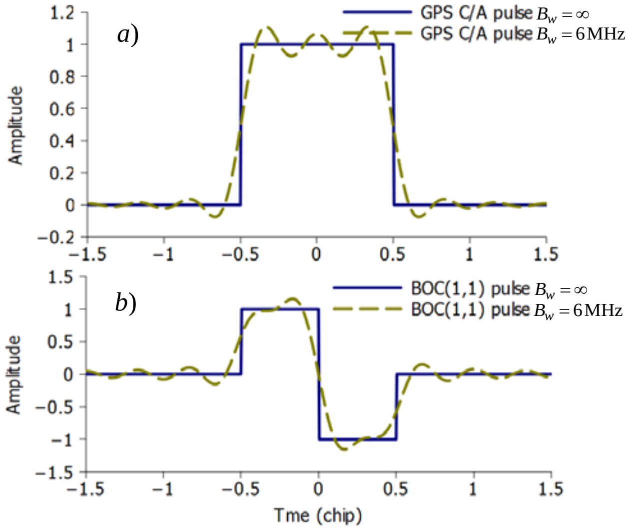

Fig. 1. a) GPS C/A and b) BOC(1,1) pulse in infinite-bandwidth and band-limited case

An expression for the power spectrum of GPS C/A periodic PRN can be written as

$$S_{\mathrm{GPSC/A}}(f) = \frac{1}{N_c^2} \Bigg(\delta(2\pi f) + \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} (N_c + 1)\, \mathrm{sinc}^2\left(\frac{m\pi}{N_c}\right) \delta\left(2\pi f + m \frac{2\pi f_c}{N_c}\right) \Bigg), \tag{6}$$

for N_c = 1023 [1,3]. The power spectrum density of the BOC(f_{sc}/f_{ref},f_c/f_{ref}) centered at the origin can be written as

$$S_{\mathrm{BOC(-)}}(f) = \begin{cases} f_c \left(\dfrac{\tan\left(\dfrac{\pi f}{2 f_{sc}}\right) \sin\left(\dfrac{\pi f}{f_c}\right)}{\pi f} \right)^2, & \text{if } \dfrac{2 f_{sc}}{f_c} \text{ is even} \\ f_c \left(\dfrac{\tan\left(\dfrac{\pi f}{2 f_{sc}}\right) \cos\left(\dfrac{\pi f}{f_c}\right)}{\pi f} \right)^2, & \text{if } \dfrac{2 f_{sc}}{f_c} \text{ is odd} \end{cases} \tag{7}$$

where f_{ref} = 1.023 MHz [3].

Fig. 2 shows power spectrum density for GPS C/A and Galileo BOC(1,1) spreading signal. From Fig. 2 it can be seen that the BOC(1,1) signal spectrum is symetric split spectrum with two main lobes shifted from the carrier frequency by the amount equal to the subcarrier frequency.

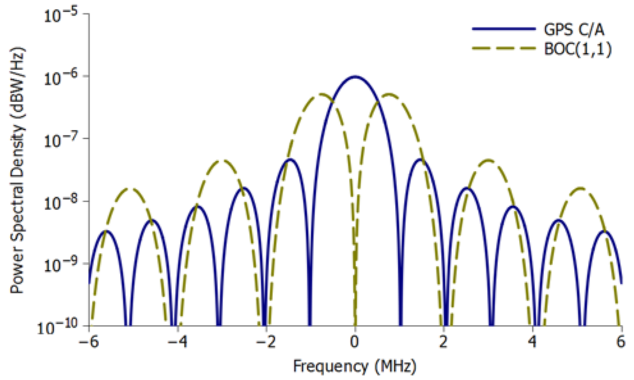

Fig. 2. GPS C/A and Galileo BOC(1,1) spectrum

B. Received signal

The received signal $r(t)$ from one satellite, in multipath environment, is composed of M paths, where one is the LOS signal and the others are reflected rays of the LOS signal. All additional sources of interference are set into a single additive Gaussian noise term, $v(t)$. After carrier removal and filtering, received signal $r(t)$ can be written as [17,23,25]

$$r(t) = a_0 q(t-\tau_0) e^{j\phi_0} + \sum_{m=1}^{M-1} a_m q(t-\tau_m) e^{j\phi_m} + v(t), \quad (8)$$

where a_m is the amplitude of the m-th path, ϕ_m is the phase of the m-th path and τ_m is the channel delay introduced by the m-th path.

C. Problem formulation

Here, we assume that the parameters of the received signal (a_m, τ_m, ϕ_m) are slowly varying, almost constant, during selected observation period. Let us define $\mathbf{a}(t) \in C^{M\times 1}$ and $\boldsymbol{\tau}(t) \in R^{M\times 1}$ as vectors containing complex amplitudes and time delays of the LOS signal and the multipath signals, respectively. The complex vector $\mathbf{a}(t) = \left[a_1(t)e^{j\phi_1(t)} \;\; \ldots \;\; a_M(t)e^{j\phi_M(t)}\right]^T$ is defined. A vector of the delayed signal components is defined as, $\mathbf{q}(t,\boldsymbol{\tau}) = [q(t-\tau_1) \;\; \ldots \;\; q(t-\tau_M)]$, where $\mathbf{q}(t,\boldsymbol{\tau}) \in C^{1\times M}$. So, the multipath signal model that is given in equation (8) can be expressed in the vector form as [23,25]

$$r(t) = \mathbf{q}(t,\boldsymbol{\tau})\mathbf{a}(t) + v(t). \quad (9)$$

Suppose that L samples of the signal are taken with a sampling interval T_s satisfying the Nyquists criterion. Then the sampled data in the k-th correlation period (period of waveform sampling and stacking) can be expressed as

$$\mathbf{z}(k) = \mathbf{Q}(k,\boldsymbol{\tau})\mathbf{a}(k) + \mathbf{v}(k), \quad (10)$$

where matrix $\mathbf{Q}(k,\boldsymbol{\tau}) \in C^{L\times M}$ is matrix containing L samples of delayed narrowband envelopes of LOS and multipath signals. The received signal and the white noise are expressed as $\mathbf{z}(k)$, $\mathbf{v}(k) \in C^{L\times 1}$, respectively [23,25].

D. ML estimation

According to maximum likelihood estimation theory, when the noise is white, the best estimates of parameters are those values that maximize following likelihood function [25]

$$p(\mathbf{z}\,|\,\boldsymbol{\tau},\mathbf{a}) = (2\pi)^{-\frac{L}{2}} |\mathbf{S}|^{-\frac{1}{2}} \exp\Big(-\tfrac{1}{2}\big(\mathbf{z}(k) - \mathbf{Q}(k,\boldsymbol{\tau})\mathbf{a}(k)\big)^H \mathbf{S}^{-1}\big(\mathbf{z}(k) - \mathbf{Q}(k,\boldsymbol{\tau})\mathbf{a}(k)\big)\Big), \quad (11)$$

where $\mathbf{S}$ is noise covariance. The minimum of log-likelihood function $\ell(\boldsymbol{\tau},\mathbf{a}) = \ln p(\mathbf{z}\,|\,\boldsymbol{\tau},\mathbf{a})$ can be found by setting the derivatives $\partial\ell \setminus \partial\mathbf{a}$, $\partial\ell \setminus \partial\boldsymbol{\tau}$ to zero. It is easy to prove that, for the fixed $\boldsymbol{\tau}$, global minimum is attained at

$$\hat{\mathbf{a}}(k) = \left(\mathbf{z}(k)^H \mathbf{Q}(k,\boldsymbol{\tau})\left(\mathbf{Q}(k,\boldsymbol{\tau})^H \mathbf{Q}(k,\boldsymbol{\tau})\right)^{-1}\right)^T. \quad (12)$$

Hence, the log-likelihood function, with $\boldsymbol{\tau}$ as parameter, can be written as

$$\ell(k,\boldsymbol{\tau}) = r_{\tilde{z}\tilde{z}}(k) - \mathbf{R}_{\tilde{z}\tilde{Q}}(k,\boldsymbol{\tau})\mathbf{R}_{\tilde{Q}\tilde{Q}}^{-1}(k,\boldsymbol{\tau})\mathbf{R}_{\tilde{z}\tilde{Q}}^{H}(k,\boldsymbol{\tau}), \quad (13)$$

where cross-correlation and auto-correlation matrices are defined as

$$\begin{aligned}&\hat{r}_{\tilde{z}\tilde{z}}(k) = \tfrac{1}{L}\mathbf{z}(k)^H\mathbf{z}(k); \quad \mathbf{R}_{\tilde{z}\tilde{Q}}(k,\boldsymbol{\tau}) = \tfrac{1}{L}\mathbf{z}^H(k)\mathbf{Q}(k,\boldsymbol{\tau})\\ &\mathbf{R}_{\tilde{Q}\tilde{z}}(k,\boldsymbol{\tau}) = \mathbf{R}_{\tilde{z}\tilde{Q}}^H(k,\boldsymbol{\tau}); \quad \mathbf{R}_{\tilde{Q}\tilde{Q}}(k,\boldsymbol{\tau}) = \tfrac{1}{L}\mathbf{Q}^H(k,\boldsymbol{\tau})\mathbf{Q}(k,\boldsymbol{\tau})\end{aligned}, \quad (14)$$

The superscript H refers to Hermitian transpose or conjugate transpose of complex matrices.

ML estimates of the time-delay and amplitude vector from equation (10) are obtained using following equations [23,25]

$$\begin{aligned}&\hat{\boldsymbol{\tau}}(k) = \min_{\boldsymbol{\tau}(k)}\{\ell(k,\boldsymbol{\tau})\}\\ &\hat{\mathbf{a}}(k) = \left(\mathbf{R}_{\tilde{z}Q}(k,\boldsymbol{\tau})\mathbf{R}_{\tilde{Q}Q}^{-1}(k,\boldsymbol{\tau})\right)^T\Big|_{\boldsymbol{\tau}=\hat{\boldsymbol{\tau}}(k)}\end{aligned}. \quad (15)$$

Fig. 3 shows auto-correlation functions for GPS C/A and BOC(1,1) signals in the band-limited case with a selected bandwidth of 6 MHz.

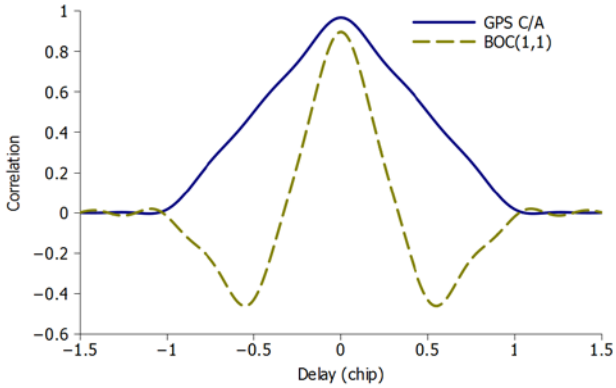

Fig. 3. GPS C/A and BOC(1,1) signal auto-correlation in the band-limited case

III. Multipath Estimating Delay Lock Loop (MEDLL)

When estimating the parameters of log-likelihood function, $\ell(\boldsymbol{\tau},\mathbf{a})$, in the situation when M-1 multipath components are present in signal, following system of equations can be used [14,15]

$$[\hat{\boldsymbol{\tau}}(k)]_m = \max_{\tau(k)} \Re\left\{\left(\mathbf{R}_{\tilde{Q}z}(k,\boldsymbol{\tau}) - \sum_{\substack{i=0\\ i\neq m}}^{M} [\hat{\mathbf{a}}(k)]_i \left[\mathbf{R}_{\tilde{Q}Q}\left(k,\boldsymbol{\tau}-[\hat{\boldsymbol{\tau}}(k)]_i\right)\right]_{i,i}\right) e^{-j\arg[\hat{\mathbf{a}}(k)]_m}\right\},$$

$$[\hat{\mathbf{a}}(k)]_m = \mathbf{R}_{\tilde{Q}z}(k,\boldsymbol{\tau}) - \sum_{\substack{i=0\\ i\neq m}}^{M} [\hat{\mathbf{a}}(k)]_i \left[\mathbf{R}_{\tilde{Q}Q}\left(k,\boldsymbol{\tau}-[\hat{\boldsymbol{\tau}}(k)]_i\right)\right]_{i,i}. \quad (16)$$

$$m = 0,\ldots,M-1$$

First equation in (16) says that for one signal component, estimated $\hat{\boldsymbol{\tau}}(k)$ is found in the maximum of the cross-correlation function when influence of the other signal components is removed. In the same manner, for the fixed $\hat{\boldsymbol{\tau}}(k)$, complex amplitude is found when influence of the other signal components is removed. When time-delay for one signal component is computed, the time-delays of the other components are not known in advance, so the following iterative algorithm will be used:

MEDLL ALGORITHM
(in the case of LOS signal and one multipath component)

1. The correlation function $\mathbf{R}_0(\boldsymbol{\tau})$ is set to $\mathbf{R}_{\tilde{Q}z}(k,\tau)$.

2. Complex amplitude $[\hat{\mathbf{a}}(k)]_0$ is found for the largest peak of the correlation $\mathbf{R}_0(\boldsymbol{\tau})$, while $[\hat{\boldsymbol{\tau}}(k)]_0$ is calculated as a maximum of the spline-interpolated correlation, $\mathbf{R}_0(\boldsymbol{\tau})$, using Newton-Raphson method.

3. Using the calculated parameters $[\hat{\mathbf{a}}(k)]_0$, $[\hat{\boldsymbol{\tau}}(k)]_0$ correlation is subtracted from $\mathbf{R}_{\tilde{Q}z}(k,\tau)$ to obtain a 2nd correlation peak $\mathbf{R}_1(\boldsymbol{\tau})$, by the expression $\mathbf{R}_1(\tau) = \mathbf{R}_{\tilde{Q}z}(k,\boldsymbol{\tau}) - [\hat{\mathbf{a}}(k)]_0\left[\mathbf{R}_{\tilde{Q}Q}\left(k,\boldsymbol{\tau}-[\hat{\boldsymbol{\tau}}(k)]_0\right)\right]_{0,0}$.

4. Complex amplitude $[\hat{\mathbf{a}}(k)]_1$ is found for the largest peak of the correlation $\mathbf{R}_1(\boldsymbol{\tau})$, while $[\hat{\boldsymbol{\tau}}(k)]_1$ is calculated as a maximum of the spline-interpolated correlation, $\mathbf{R}_1(\boldsymbol{\tau})$, using Newton-Raphson method.

5. Using the calculated parameters $[\hat{\mathbf{a}}(k)]_1$, $[\hat{\boldsymbol{\tau}}(k)]_1$, correlation is subtracted from $\mathbf{R}_{\tilde{Q}z}(k,\tau)$ to obtain a 1st correlation peak $\mathbf{R}_0(\boldsymbol{\tau})$, by the expression $\mathbf{R}_0(\tau) = \mathbf{R}_{\tilde{Q}z}(k,\boldsymbol{\tau}) - [\hat{\mathbf{a}}(k)]_1\left[\mathbf{R}_{\tilde{Q}Q}\left(k,\boldsymbol{\tau}-[\hat{\boldsymbol{\tau}}(k)]_1\right)\right]_{1,1}$.

6. Steps from 2 to 5 are repeated until the predefined stopping criterion is met.

Figs. 4 and 5 are showing multipath error envelopes for GPS C/A signal and Galileo BOC(1,1) signal, respectively, when the narrow EML delay lock loop (with 0.1 chip correlator spacing) and the MEDLL are used. The multipath error envelopes are representing change of the LOS signal ranging error in dependence of the multipath signal delay when multipath component is in the constructive phase ($\phi_1 - \phi_0 = 0°$, solid lines) and in the destructive phase ($\phi_1 - \phi_0 = 180°$, dashed lines). It can be seen that LOS signal ranging error is significantly lower for the MEDLL, regardless of signal used.

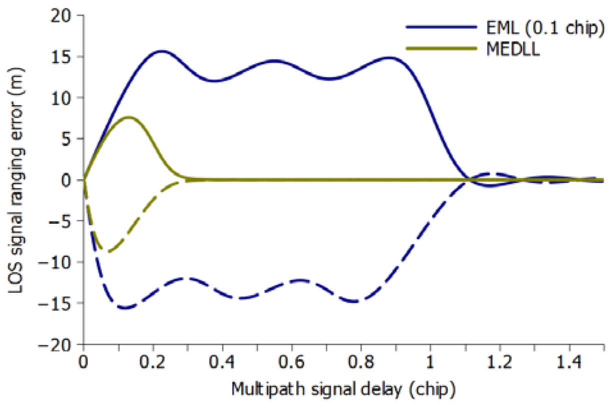

Fig. 4. Multipath error envelopes for the GPS C/A signal

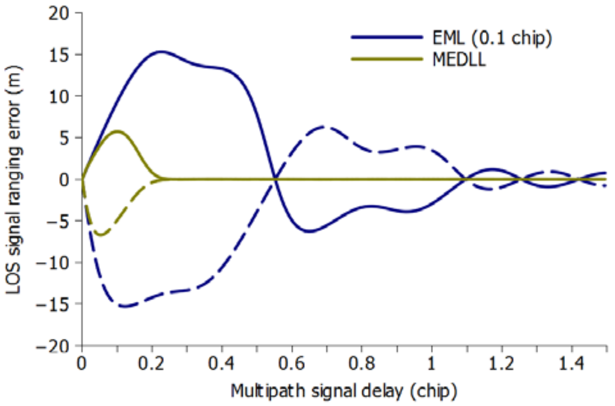

Fig. 5. Multipath error envelopes for the Galileo BOC(1,1) signal

IV. PARTICLE FILTER

A. Particle filter theory

The particle filter aims to estimate, recursively in time, state $\mathbf{x}(k) \in C^{2M\times 1}$, based only on the observed data $\mathbf{z}(k) \in C^{L\times 1}$ at time index *k*. Particle filter is based on Bayesian estimation that follows the posterior density function $p\left(\mathbf{x}(k) \middle| \mathbf{Z}_k\right)$ which contains information about the state $\mathbf{x}(k)$, where $\mathbf{Z}_k = \left\{\mathbf{z}(1),...,\mathbf{z}(k)\right\}$ is set of observations until present time [26].

Nonlinear, non-Gaussian state space model can be written as follows

$$\mathbf{x}(k) = \mathbf{f}\left(\mathbf{x}(k-1), \mathbf{v}(k)\right), \tag{17a}$$

$$\mathbf{z}(k) = \mathbf{h}\left(\mathbf{x}(k-1)\right) + \mathbf{w}(k), \tag{17b}$$

where equation (17a) represents state equation of the discrete-stochastic system defining its dynamical behavior. Second equation is called measurement equation and it returns observed data.

The particle filter approximates the probability density $p\left(\mathbf{x}(k)\middle|\mathbf{Z}_k\right)$ by a large set of *P* particles, $\mathbf{x}^i$, $i = 1,...,P$ where each particle has an assigned relative weight, $w^i(k)$, so that sum of all weights equals one. The location and weight of each particle reflects the value of the density in the region of the state space. The particle filter updates the particle location and the corresponding weights recursively with each new observation. The filtering density, $p\left(\mathbf{x}(k)\middle|\mathbf{Z}_k\right)$, and the one step prediction density $p\left(\mathbf{x}(k+1)\middle|\mathbf{Z}_k\right)$ are given by a measurement update according to following equations

$$p\left(\mathbf{x}(k)\middle|\mathbf{Z}_k\right) = \frac{p\left(\mathbf{z}(k)\middle|\mathbf{x}(k)\right) p\left(\mathbf{x}(k)\middle|\mathbf{Z}_{k-1}\right)}{p\left(\mathbf{z}(k)\middle|\mathbf{Z}_{k-1}\right)}, \tag{18}$$

$$p\left(\mathbf{z}(k)\middle|\mathbf{Z}_{k-1}\right) = \int p\left(\mathbf{z}(k)\middle|\mathbf{x}(k)\right) p\left(\mathbf{x}(k)\middle|\mathbf{Z}_{k-1}\right) d\mathbf{x}(k), \tag{19}$$

and the time update or prediction according to

$$p\left(\mathbf{x}(k+1)\middle|\mathbf{Z}_k\right) = \int p\left(\mathbf{x}(k+1)\middle|\mathbf{x}(k)\right) p\left(\mathbf{x}(k)\middle|\mathbf{Z}_k\right) d\mathbf{x}(k). \tag{20}$$

The recursion is initiated with known distribution $p\left(\mathbf{x}(0)\middle|\mathbf{Z}_{-1}\right) = p\left(\mathbf{x}(0)\right)$, where $\mathbf{Z}_{-1}$ is set without observations [26].

The likelihood $p\left(\mathbf{z}(k)\middle|\mathbf{x}(k)\right)$ is calculated from equations (11) and (17b) using the known measurement noise probability density function. This function is used for calculation of importance weights $w_i = p\left(\mathbf{z}(k)\middle|\mathbf{x}^i(k)\right)$. The aim is to approximate posterior density $p\left(\mathbf{x}(k+1)\middle|\mathbf{Z}_k\right)$, with a sum of weighted delta-Dirac functions [26]

$$p\left(\mathbf{x}(k)\middle|\mathbf{Z}_k\right) \approx \sum_{i=1}^{P} \tilde{w}_k^i \delta\left(\mathbf{x}(k) - \mathbf{x}^i(k)\right), \tag{21}$$

where the normalized importance weights are defined as

$$\tilde{w}_i(k) = w_i(k) \sum_{j=1}^{P} w_j(k), \quad i = 1,...,P. \tag{22}$$

This approach, called Sequential Importance Sampling (SIS) often leads to divergence, where all the weights are tending to zero. Using selection or resampling step this problem can be handled [26]. The main idea behind the resampling step is to discard particles with small weights and to multiply particles with large weights, particles that corresponding to large likelihoods. This is done by drawing a new set of particles, with replacement from the old particles. A suitable measure of degeneracy of the algorithm is the effective sample size N_{eff}. This value cannot be exactly calculated, so an estimate $\hat{N}_{eff}$ is used

$$\hat{N}_{eff} = \frac{1}{\sum_{j=1}^{P} \tilde{w}_j(k)}. \tag{23}$$

When $\hat{N}_{eff}$ is smaller than a certain user defined threshold, N_{th}, we apply the resampling step in order to decrease the variance of the importance weights [26].

Once we have approximated posterior density we can either determine particle that maximizes it, the so called *maximum a-posteriori* (MAP) estimate, or we can find the expectation, equivalent to the *minimum mean square error* (MMSE) estimate.

B. Particle filter algorithm

The complex-valued state vector $\mathbf{x}(k)$ contains delays for the LOS signal and the multipath signals and their corresponding complex amplitudes. In section II is said that for known delays it is possible to calculate amplitudes, so we are simplifying state vector with vector $\boldsymbol{\tau}(k)$. This vector contains only time delays for the LOS signal and the multipath signals and can be written as $\boldsymbol{\tau}(k) = \left[\tau_0(k) \quad \tau_1(k) \quad \dots \quad \tau_{M-1}(k)\right]^T$.

1. **Initialization**. After the acquisition, the LOS signal delay uncertainty is in range $\left[-T_u, T_u\right]$, while for the multipath signal, delay is mostly in a range $\left[\tau_0, 2T_c\right]$. So,

the particles for the LOS signal and the multipath signal delay can be initialized as

$$\begin{aligned} &\tau_0^i(0) \sim U(-T_u, T_u) \\ &\tau_m^i(0) \sim U\left(\tau_0^i, 2T_c\right), \quad i=1,\ldots,P, \quad m=1,\ldots,M-1 \end{aligned} \tag{24}$$

with the weights that are equal.

2. **Importance Sampling.** Since the likelihood function is the Gaussian distribution, it is quite reasonable to propose the Gaussian importance function for particle generation. Here, for the LOS signal, importance function is realized as the Gaussian distribution with mean in the previous MAP estimate for LOS signal delay and with a variance calculated using posterior particles. Similarly, the multipath signal delay is generated using the Gaussian distribution, but in the way that newly generated values for the delay of multipath signals in the particle are larger than the delay of the LOS signal. Thus,

$$\begin{aligned} &\tau_0^i(k) \sim N\left(\hat{\tau}_0^{MAP}(k), \sigma_0^2(k)\right) \\ &\tau_m^i(k) \sim \tau_0^i(k) + \left| N\left(\hat{\tau}_m^{MAP}(k) - \tau_0^i(k), \sigma_m^2(k)\right)\right|, \\ &\qquad i=1,\ldots,P, \; m=1,\ldots,M-1 \end{aligned} \tag{25}$$

3. **Weight update and estimation**. For every particle, complex-valued signal replica is generated and $\mathbf{Q}(k,\boldsymbol{\tau})$ matrix is formed. Weights are calculated using equations (13) and (14) and then normalized with equation (18). After that, MAP estimate that maximizes posterior density based on equation (15) is found. Also, the a priori error covariance of the delay, $\boldsymbol{\Sigma}(k)$, must be calculated in order to measure estimated accuracy of the time delay. Following equation is used

$$\boldsymbol{\Sigma}(k) \approx \sum_{i=1}^{P} \tilde{w}_i(k)\left(\boldsymbol{\tau}^i(k) - \hat{\boldsymbol{\tau}}^{MAP}(k)\right)\left(\boldsymbol{\tau}^i(k) - \hat{\boldsymbol{\tau}}^{MAP}(k)\right)^T, \tag{26}$$

with $\sigma_m^2 = \left[\boldsymbol{\Sigma}(k)\right]_{m,m}$.

4. **Resampling. A**ccording to the equation (23) value of $\hat{N}_{eff}$ is calculated. If value of $\hat{N}_{eff}$ is less then then the value of threshold N_{th}, multinomial resampling (see [26]) is performed.

V. Simulation and Results

The simulated GPS C/A and BOC(1,1) signals are composed of a LOS signal and one multipath component (M=2). Both signals are generated on intermediate frequency of $f_{IF} = 4.092$ MHz with sampling frequency $f_s = 16.368$ MHz. Before carrier removal and spreading, signal is filtered with 6 MHz bandwidth filter. Accepted relative amplitude between LOS signal and multipath signal is α = 0.5 while delay uncertainty T_u is $0.1T_c$. Selected signal-to-noise ratio (SNR) is -20 dB. It is supposed that phase difference between the LOS signal and the multipath signal is 10° while time-delay of multipath component is 0.2 chip.

In case of the GPS C/A signal results are obtained for two different correlation periods. The first period is 1 ms and it corresponds to duration of the GPS C/A spreading sequence, while the second period is prolonged on 4 ms. On the other hand, correlation period for BOC(1,1) signal is 4 ms and it corresponds to duration of the 4092 chips long BOC spreading sequence (f_c=1.023 MHz). The estimation results in the case of the MEDLL and the PF (for P=1000 particles) are shown on Figs. from 6 to 11.

Figs. 6 and 7 are showing estimated GPS C/A LOS signal delay and GPS C/A multipath signal delay, respectively, for the MEDLL and the PF filter. On the Fig. 6 can be seen that the MEDLL algorithm has less variance than the PF filter, but the MEDLL is introducing some bias with regard to the LOS signal delay.

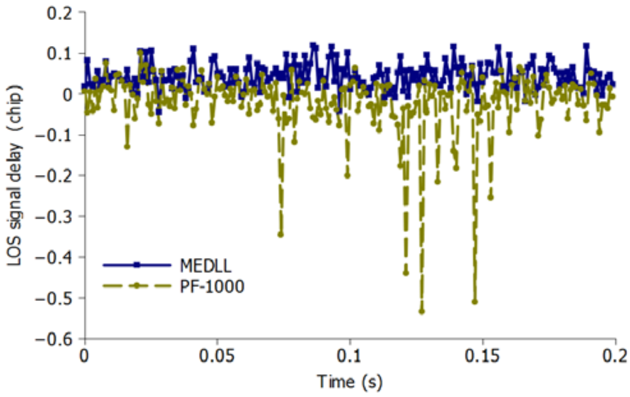

Fig. 6. Estimated GPS C/A LOS signal delay in time with correlation period of 1 ms

As can be seen on Fig. 7, the estimated GPS C/A multipath signal delay in case of the PF is much more precise then the MEDLL algorithm.

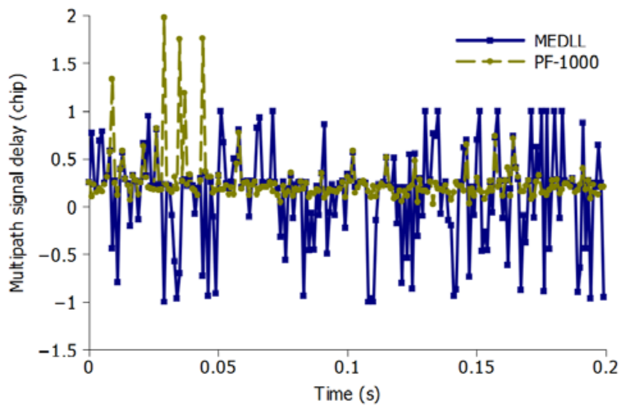

Fig. 7. Estimated GPS C/A multipath signal delay in time with correlation period of 1 ms

As shown on Fig. 8, for the correlation period of 4 ms, the estimated delay of the GPS C/A LOS signal component in the case of the PF is somewhat more precise compared to the estimation obtained using the MEDLL algorithm. Here, like on Fig. 6, the MEDLL algorithm is introducing some bias with regrad to the delay of the LOS signal component.

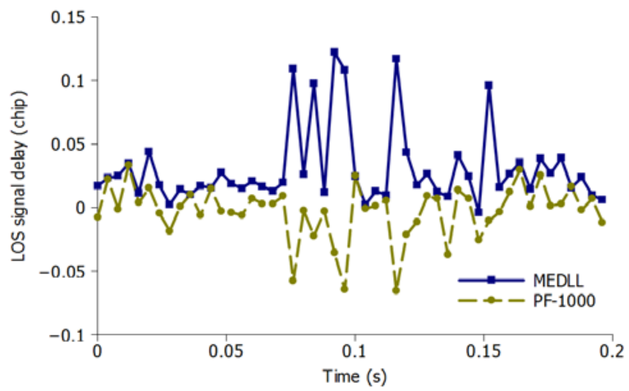

Fig. 8. Estimated GPS C/A LOS signal delay in time with correlation period of 4 ms

On Fig. 9 can be seen, that the estimation of the GPS C/A multipath signal component in the case of the MEDLL and the PF filter is almost the same with the exception of few peaks for the MEDLL algorithm.

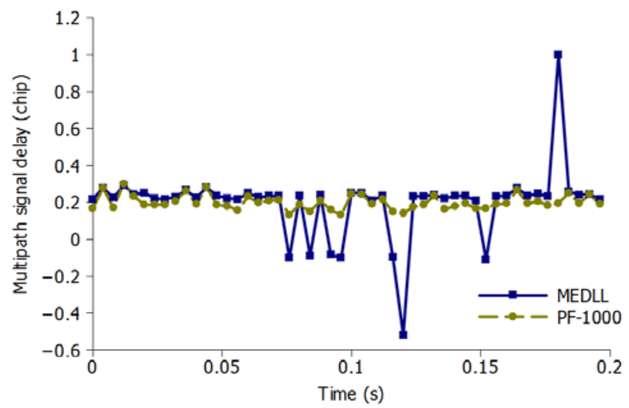

Fig. 9. Estimated GPS C/A multipath signal delay in time with correlation period of 4 ms

Figs. 10 and 11 are showing estimation for the LOS and multipath signal delay when the Galileo BOC(1,1) signal is used. As can be seen on the both figures, estimation does not favour any of the implemented algorithms, and estimation in the case of the MEDLL algorithm is similar to the estimation of the PF filter.

The MEDLL algorithm iterations are stopped when $[\hat{\tau}(k)]_0$ changes 0.1 ns between two successive iteration steps, or after 10 successive steps, whichever occurs earlier. The number of complex samples is 61. These samples are equally spread on the interval from -2T_c to 2T_c.

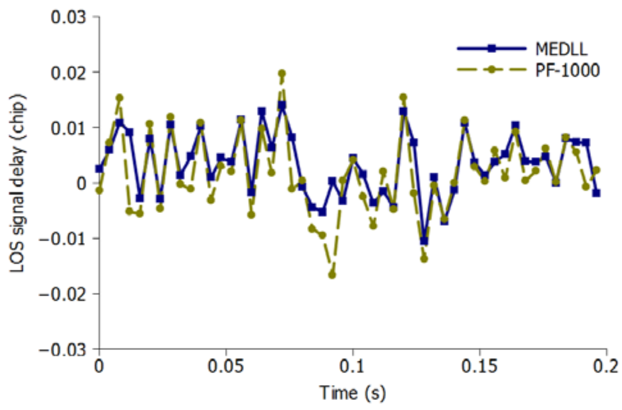

Fig. 10. Estimated BOC(1,1) LOS signal delay

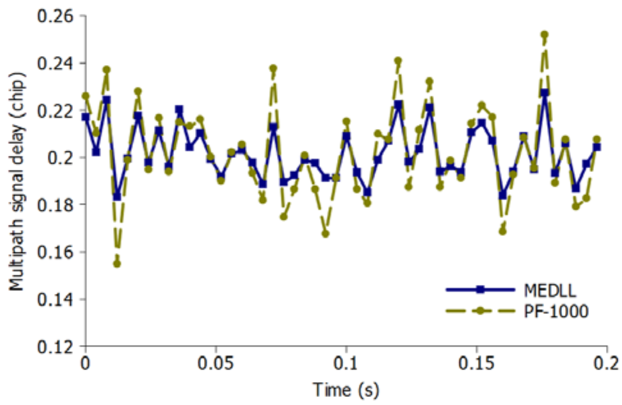

Fig. 11. Estimated BOC(1,1) multipath signal delay

IV. Conclusion

In this paper, two algorithms for the multipath mitigation have been presented. The simulation environment is set and composite GPS and Galileo signals are created. Using simulated signals, estimation efficiency of the algorithms is mutually compared. From the analysis it can be concluded that the particle filter, with large number of particles, is more precise then the MEDLL algorithm. This is primarily true when estimating delay of the GPS C/A signal with correlation periods of 1 ms and 4 ms. With larger correlation period MEDLL estimation would be closer to the particle filter estimation.

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